COVID-19 in air suspensions

Daniel A. Stariolo[1]

[1]*Departamento de Física, Universidade Federal Fluminense and National Institute of Science and Technology for Complex Systems, Av. Litorânea s/n, Campus Praia Vermelha, 24210-346 Niterói, RJ, Brazil*

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We present a theoretical study of the motion of virus particles as they reach quiet air after a person breaths, speaks, sneezes or coughs. Because the viruses are expelled mainly attached to small droplets, with diverse sizes and weights, and the external environmental conditions can also be diverse, the subsequent motion cannot be described by a unique model. Computing the time of decay to the ground and the distance travelled by a droplet using a simple free fall model, we obtained a typical horizontal distance in the range between 1 to 3 meters from the emitter, with a falling time of less than a second, similar to initial previsions of falling distance. For the smallest droplets we employ a model of motion in a viscous medium, obtaining that isolated viruses could remain suspended in quiet air for more than a month, while droplet nuclei can remain for several hours, in agreement with recent experimental results on virus stability in aerosols.

Introduction

With the advance of the Covid-19 pandemic there is an ongoing discussion on the proper use of masks as a safeguard against contagion. The initial advise from the World Health Organization was that only confirmed infected people should wear masks, in order to prevent transmission of the virus by sneezing and coughing [1]. But the high rate of propagation of the virus in different environments around the world has raised a concern about this initial prescription. In particular, the fact that most of infected people go without developing symptoms make them potential transmitter of the desease. Together with restrictions on human circulation and social distancing, other important protective measure can be the generalized use of masks in public places, like supermarkets and hospitals.

Our aim is to present a pedagogical discussion of the motion of the virus in suspended aerosols. To this aim, we will consider the Covid-19 virus a single particle of mass m_C. When breathing, sneezing, talking or coughing, droplets with a viral charge are expelled from the mouth of an infected person, forming what is called aerosols. The droplets dispersed in the sorrounding air do not interact between them. Aerosols with a viral charge can be suspended in the air for a given amount of time. The main characteristics of the particles which determine the time of suspension in the air are the particle mass and size. In general, there is a distribution of particle masses and sizes in an aerosol. As a consequence of gravity, heavier particles will fall down at a faster rate than lighter particles. Nevertheless, recent reports show that the Covid-19 can remain suspended and active in aerosols for hours [2] making them potentially dangerous for the transmission of the desease to healthy people breathing around.

We will present some results based on simple physical models for the propagation of the particles of virus once they come out from the mouth of an infected person. First, we will consider a virus that is ejected from the mouth as a free falling object under the only influence of gravity. We will show that this is not a realistic situation, but nevertheless it is useful as a reference for other, more complex approximations. Furthermore, the results for this model predict that the virus should settle down on the ground after travelling approximately 1 to 2 meters, as suggested by different sources and transmitted in the general media [3]. Then, we will present some estimates of the air interparticle distance as compared with the size of the coronavirus and of larger droplets. This will suggest us that the virus falls down in a viscous medium, the air. Because the droplets in aerosols where the virus can be found show a distribution of sizes and masses, we will show simple estimations of the rate and time of decay for typical sizes of the particles. This analysis leads us to conclude that typical droplets containing virus may remain suspended in quiet air for more than an hour.

A droplet in free fall

Free fall is the simplest model for the falling of a particle under the influence of gravity. It is assumed that there are no collisions with other particles during the trajectory. The position and velocities of the particles are given by Newton's equations of motion for a free fall [4]:

$$\begin{aligned} x(t) &= x_0 + v_{0x} t \\ v_x(t) &= v_{0x} \\ y(t) &= y_0 + v_{0y} t + \frac{1}{2} g\, t^2 \\ v_y(t) &= v_{0y} + g t, \end{aligned} \tag{1}$$

where x and y refer to horizontal and vertical directions respectively, (x_0, y_0) are the initial horizontal and vertical position coordinates of the particle, (v_{0x}, v_{0y}) are the corresponding components of the initial velocity vector, and $g = -9,8\,m/s^2$ is the acceleration of gravity (the minus sign reflects that distances grow positive from bottom to top, and gravity points downward in this reference frame). Here, we will be interested in knowing how far a particle can travel, once it leaves the mouth of a person with an initial velocity vector in the horizontal direction, given by $(v_{0x}, v_{0y} = 0)$. The initial position of the particle will be set to $(x_0 = 0, y_0 = h)$, where h is the height at which the mouth is. Within these conditions, from the third equation from (1) we can obtain the time to reach the floor $(y = 0)$ from an initial height $y_0 = h$:

$$t(h) = \sqrt{\frac{2h}{g}}. \tag{2}$$

Then, for example, a particle in free fall from a height $h = 1.7\,m$ will reach the floor in $t(1.7\,m) = 0.59\,s$. The horizontal distance it can travel is then obtained from the first of equations (1). It depends on the initial horizontal velocity v_{0x}. We chose two representative cases, extracted from an experimental study reported in [5]. We considered the maximum reported velocities, a conservative choice. The maximal initial velocity of a particle when talking or breathing was reported to be $v_1 \approx 1.5\,m/s$, while the maximal initial velocity

$h[m]$	$t_h[s]$	$x_{v_1}(t_h)[m]$	$x_{v_2}(t_h)[m]$
1.5	0.55	0.83	2.77
1.6	0.57	0.86	2.86
1.7	0.59	0.88	2.95
1.8	0.61	0.91	3.03

TABLE I. Times of decay to the floor and horizontal distances for talking or breathing (x_{v_1}) and sneezing or coughing (v_{v_2}) for four initial heights, computed with a free fall model.

when sneezing or caughing was $v_2 \approx 5.0\,m/s$. Although in the report these corresponded to maximal speed, i.e. the scalar value of the velocity vector, we will consider them to be maximal representative of horizontal velocities, which we assumed in this case. Some values of time of decay and travel distance to the floor are shown in Table I.

As we can see in Table I, the times for a particle in free fall to settle in the ground from the typical height of a person is less than a second. The estimates of the horizontal distance travelled by the virus is less than $1\,m$ when talking or breathing, and at most $3\,m$ from sneezing or coughing. These estimates roughly agree with others suggesting that keeping a distance of around $2\,m$ between people could be enough to represent a secure situation [3]. Nevertheless, as will be shown in the next section, the free fall approximation is not accurate for the decay of small particles of the size of a virus in the air, and does not agree with studies that show that droplets in aerosols may stay suspended in the air for more than an hour [2, 6]. In the next section we will show that a better model for small particles of the order of $1\,\mu m$ is to consider that they flow in a viscous medium, the air.

Particle flow in a viscous medium

The density of dry air at $T = 20^oC$ and $1\,atm$ is $\rho_{air} = 1.2041\,kg/m^3$ [7]. Considering the air in a cube of volume $V = l^3$, we can estimate the typical interparticle distance in air to be:

$$l \approx (m/\rho_{air})^{1/3}, \tag{3}$$

where m is the average mass of an air particle (mainly nitrogen and oxigen molecules). The average molecular mass of dry air is $M_{air} = 28.97\,g/mol$ [8]. Considering that there are $N_A = 6.022 \times 10^{23}$ particles in a mol of substance, where N_A is Avogadro's constant, then $m = M_{air}/N_A \approx 4,81 \times 10^{-26} kg$. Thus, the typical interparticle distance in air is $l \approx 3,42 \times 10^{-9}\, m$, or $l = 3.42\, nm$. To compare with, the approximate diameter of the Covid-19 virus is $d_C \approx 100\, nm$ [9]. Then, the virus will hit many particles in its falling trajectory through the air. As a better model to compute characteristics of the trajectory of the virus we can assume that it is an approximately spherical particle falling in a viscous medius, the air. Furthermore, in the vast majority of cases, the virus will come attached to small water droplets expelled when talking or coughing. The force of viscosity exerted by a fluid on a small sphere flowing through it, is given by Stokes' law [10, 11]:

$$F_d = 6\pi\, \eta\, r\, v, \tag{4}$$

where η is the viscosity, r the radius of the sphere and v the relative velocity between the fluid and the object. This upward friction force grows with time proportional to the velocity. It is opposed by the excess force between the weight and the buoyancy:

$$F_g = (\rho_s - \rho_f)\, g \frac{4}{3}\pi r^3, \tag{5}$$

where ρ_s is the density of the sphere and ρ_f that of the fluid. In this case, we will consider water spherical droplets with $\rho_s = 997\, kg/m^3$ (the density of water) and $\rho_f = \rho_{air}$ given above. Then, $\rho_s \approx 1000\, \rho_{air}$ and we can discard the density of air in F_g, which is reduced simply to the weight of the spherical droplet, $F_g \approx m_s g$. When the upward drag force F_d equals the weight of the particle, $F_d = F_g$, the droplet reaches mechanical equilibrium and, from then on, continues to fall with constant *terminal velocity* given by:

$$v_T = \mu\, m_s g, \tag{6}$$

where $\mu = 1/6\pi\eta\, r$ is the droplet mobility in the fluid. For air, the dynamical viscosity is $\eta = 18.5\, \mu Pa \cdot s = 1.85 \times 10^{-5}\, kg \cdot m^{-1} \cdot s^{-1}$. Assuming that particles have an initial vertical velocity equal to zero, the terminal velocity is the maximal velocity it attains while falling down.

First of all, lets make an estimation of the limits of validity of the "free fall" model of the previous section. The free fall model assumes that gravity, i.e. the weight, is the only relevant force during the motion of a particle. Then, as long as the weight is much larger than the drag force, the motion can be considered a free fall. Now, the weight of a spherical particle $F_g = m_s g$ can be written equivalently as $F_g = \rho_s V\, g = 4/3\,\pi r^3 g$. We note that the weight of the sphere grows proportional to its volume, i.e. as the cube of its radius. On the other hand, the drag force produced by the viscosity of air grows linearly with the radius of the sphere $F_d = 6\pi\,\eta\, r\, v$, much slower than the weight. Then, one can naively expect that the weight will quickly be much larger than the drag force as the size of the particle grows. Nevertheless, F_d also depends on the velocity, which is growing in the initial part of the motion, before it attains its terminal value. In any case, the real velocity when the particle has fallen down a height h will always be smaller than the corresponding free fall velocity v_{ff}, given by the last of equations (1) with the time given by (2), i.e.

$$v_{ff}(h) = \sqrt{2gh}. \tag{7}$$

We can set this value as an upper bound for the velocity of the particle after falling from a height h. In the present situation, considering a typical height $h = 1.7\,m$, the free fall velocity is $v_{ff} = 5.77\,m/s$. Equating the weight with the drag force computed at v_{ff} we can obtain an upper bound for the radius of the particle r_c:

$$r_c = \sqrt{\frac{9}{2}\frac{\eta}{\rho g} v_{ff}}. \tag{8}$$

When the radius $r \geq r_c$ the drag force becomes irrelevant and the free fall model is valid. For the case under consideration we obtain $r_c \approx 200\,\mu m$. Several studies reported a wide range of droplet sizes, from the order of $1\,\mu m$ up to millimiters (see. e.g. Table 1 in [12]). Droplet sizes are strongly dependent on the nature of the phenomenon, while talking, breathing, sneezing or caughing. For example, it has been found that sneezing produces larger droplets than coughing, and also that the velocity at which the droplets are expelled is considerably larger during sneezing. Other characteristics that may influence the final results reported in the literature are measurement techniques, the size of the sampled indiviuals and their health conditions. Last but not least, it is also known that droplets suffer evaporation after coming out of the respiratory tract. Then, the methodology of measurment is also important. The further from the mouth, the smaller will be the droplet size, due to evaporation. Results on typical droplet sizes in sneezing are around $\sim\, 100\,\mu m$ [12]. For this size the time of evaporation has been reported to be $\approx 1.7\,s$ [13], i.e. larger than the time for free fall of all cases considered in Table I. Then, the free fall estimates may be relevant for the motion of sneezing droplets.

Now, lets consider some cases with droplets smaller than $100\,\mu m$, such as those typically produced during talking or coughing. As we have seen, for these sizes the effects of the drag force cannot be discarded. To begin with, the smallest droplet of interest is a single Covid-19 virus. The radius of the virus is $r_C \approx 50\,nm$, and its mass $m_C \approx 10^{-18}\,kg$ [6]. Its mobility and terminal velocity in dry air will be:

$$\mu = \frac{1}{6\pi\eta r} \approx 0.58 \times 10^{11}\,s \cdot kg^{-1}$$
$$v_T = \mu\, m_C g \approx 5,68 \times 10^{-7}\,m/s. \tag{9}$$

The downward *maximal* velocity of a single virus is too small. Assuming that it began its fall with this vertical velocity at a height $h = 1.7\,m$, it will reach the ground in a time $t = h/v_T \approx 0.3 \times 10^7\,s \sim 35\,d$. Then, single viruses can float in quiet air for more than a month before settling down on the ground.

The previous case of single viruses floating freely in the air is probably not the most probable situation, nor the most dangerous from a contamination perspective, because the viral charge of the virus depends on its concentration. More relevant are droplets (mainly composed of water) with a virus charge in them. We can consider two typical cases: whole droplets as they come out from the mouth of infected people, and droplets nuclei, that are the remanent of the original droplets once the water has evaporated. The typical size of droplet nuclei when coughing is around $d_{dn} \approx 1\,\mu m$ and that of whole droplets $d_d \approx 10\,\mu m$ [14]. In fact, these values correspond to whole particle sizes, i.e. diameters. Assuming that the droplets content are mainly water, the terminal velocities and times of decay from a height $h = 1.7\,m$ for both cases are:

$$v_{dn} \approx 2.46 \times 10^{-4}\,m/s, \quad t_{dn} \approx 1.9\,h \qquad \text{for droplets nuclei,}$$
$$v_d \approx 2.46 \times 10^{-2}\,m/s, \quad t_d \approx 1.2\,mins \qquad \text{for water droplets.} \tag{10}$$

These order of magnitude estimates show that droplets nuclei of $\sim 1\,\mu m$ can remain suspended in air for around 2 hours, while droplets of $\sim 10\,\mu m$ of size can remain for around a minute. These results are compatible with recent experimental determinations of the half-life of the virus in aerosols [2, 6].

Conclusions

We have shown theoretical estimates of the motion of droplets in the air, depending on droplet size and initial velocities as the leave the mouth during talking, breathing, sneezing or coughing. Depending on the size of the droplets, which can vary in a wide range, two different models can give useful estimates of the time of persistence of the droplets in quiet air. For droplets larger than $\approx 200\,\mu m$, typical of sneezing events, a free fall model is appropriate. The maximum distance attained is around $3\,m$, in agreement with recommendations for distancing found in the press media. On the other side, for droplets of sizes less than $\approx 100\,\mu m$, the free fall model is not accurate, and a model of motion in viscous air is necessary. Our results show that typical respiratory droplets with sizes $\leq 10\,\mu m$ can remain suspended in dry quiet air from minutes to hours. The time of evaporation of water droplets of sizes as small as $10\,\mu m$ is much less than a second [13], leaving droplets nuclei. Then, we conclude that droplet nuclei, which can be suspended in dry air for hours, may be carriers of the virus in quiet air, like in a closed room or inside an aircraft. These theoretical estimates are in agreement with recent results on the stability of the Covid-19 in aerosols.

Together, these results suggest that airborne transmission of the virus may be possible and must be considered. This also points that the use of masks, even for healthy people, may be a valid way of reducing contagion of Covid-19. More experimental studies are needed in order to confirm if airborne transmission is relevant as compared with other known mechanisms, like surface mediated ones.

Besides simple diffusion in quiet air, convection is certainly another important mechanism for airborne infection. It has been reported than even in the presence of convective flows the virus could spread in a large room before it settles on surfaces or is dispersed [13].

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